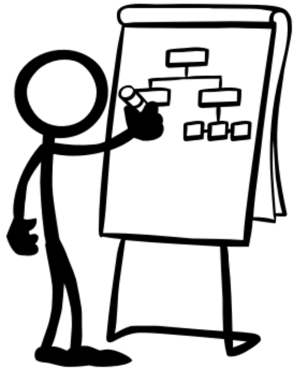
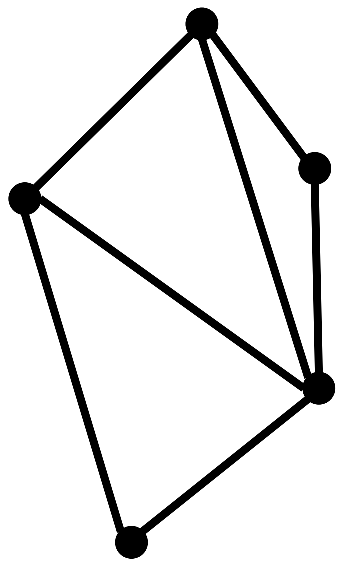
Triangulations and Related Problems

Computational Geometry – Recitation 10

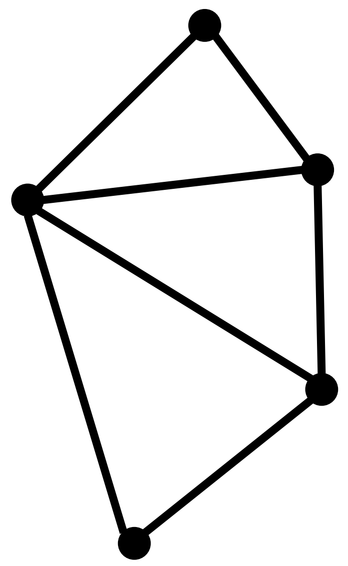


Delaunay Triangulation

- Reminder – a triangulation which maximize the minimum angle in the triangulation.



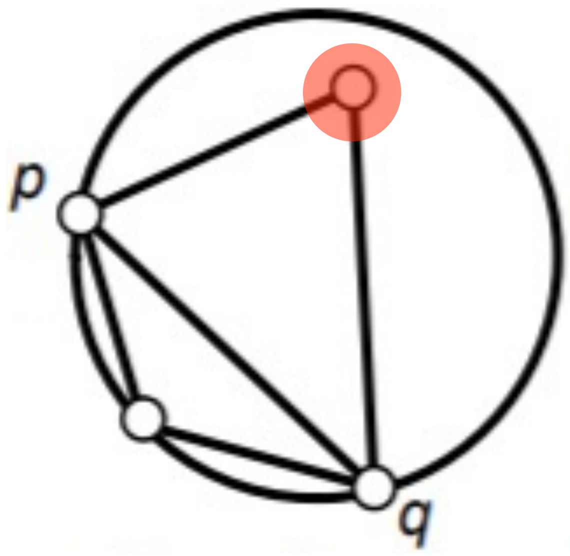
Non-Delaunay triangulation

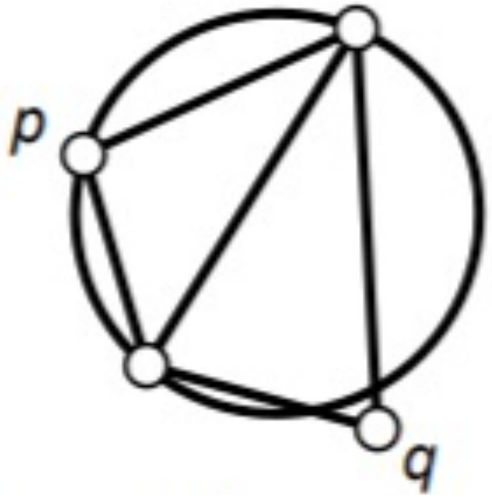


Delaunay triangulation

Delaunay Triangulation – Observations

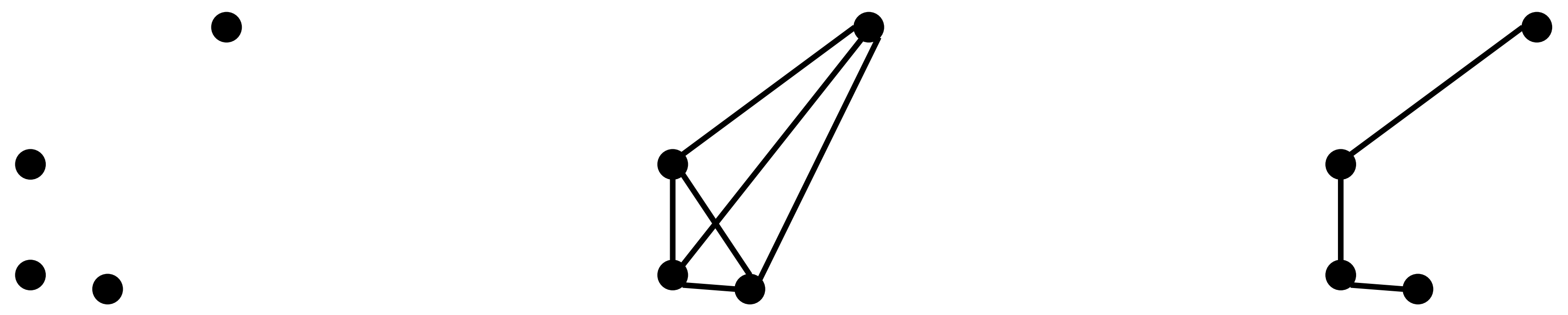
A triangle is Delaunay $\longleftrightarrow$ the circle through its vertices is empty of other sites





Other kinds of graphs - EMST

- $EMST$ (Euclidean Minimum Spanning Tree) –
  A set of edges spanning a set of points with the minimum total edge length.

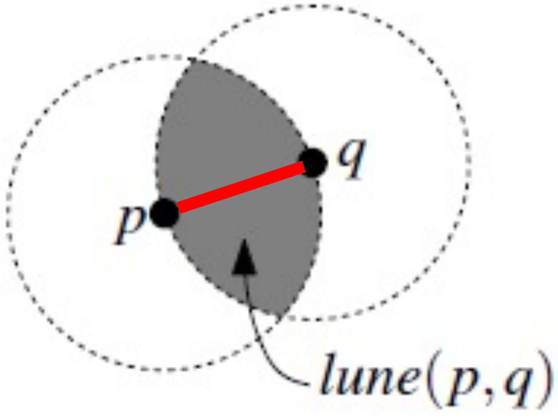


Other kinds of graphs - RNG

- RNG (Relative neighborhood graph) –
  An edge (p, q) is a part of the RNG iff

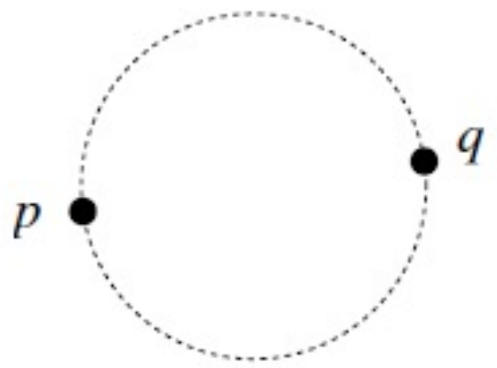
$$d(p,q) \leq \min_{r \in P, r \neq p,q} \max(d(p,r), d(r,q))$$

- <u>Claim:</u> The edge (p, q) is part of the RNG iff the *lune of* p and q is empty.
- It is easy to see from the definition.



Other kinds of graphs - GG

- GG (Gabriel Graph) –
  Two points p and q are connected by an edge of the GG iff
  the disc with diameter pq does not contain any other point of P.



Other kinds of graphs

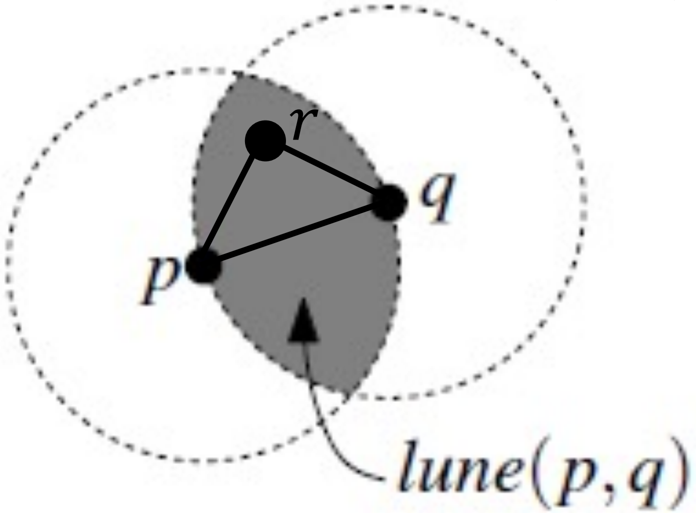
- Prove that

$$EMST \subseteq RNG \subseteq GG \subseteq DT$$

- The last relation is part of the HW.
- We will show the other two.

$EMST \subseteq RNG$

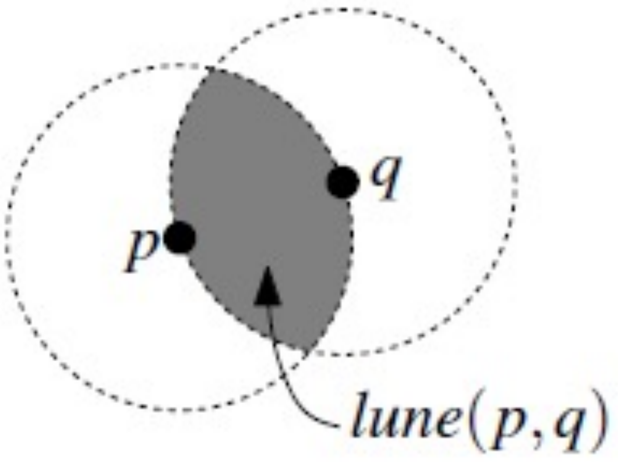
- Let $(p, q) \in EMST(P)$, and assume that $(p, q) \notin RNG(P)$
- That means that there exist point $r \in lune(p, q)$

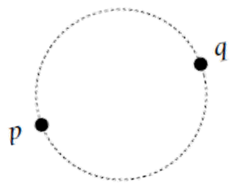


- The edge (p, q) is the largest in the circle pqr, and thus, can not be part of $EMST(P)$.
  - Recall Algo 1 MST rules

$RNG \subseteq GG$

- Reminder: $(p,q) \in RNG(P)$ iff $lune(p,q)$ is empty.
- $(p,q) \in GG(P)$ iff the the disc with diameter pq does not contain any other point of P.





- The circle is subset of the lune, and thus the claim.

Euclidean Traveling Salesman Problem

- The *traveling salesman problem* (TSP) is to compute a shortest tour visiting all points in a given point set.
  - NP-hard.

- In the Euclidean version the distances are the Euclidean distance.
- Show how to find a tour T' whose length is at most two times the optimal length.

$$|T'| \leq 2 \cdot |opt|$$

Euclidean Traveling Salesman Problem

- Claim:

The optimal tour length is longer (or equal) to the $EMST$ weight

- Proof:

Consider the graph created by the TSP tour, this graph spans the set of points and thus its weight is greater than the $EMST$ weight.

Euclidean Traveling Salesman Problem

- Claim:

The length of a DFS traversal over the $EMST$ is at most twice the length of the $EMST$

  - (and thus, at most twice the length of the optimal tour).

- Proof:
  Each edge is traversed at most twice.
- TSP Approximation algorithm: Find the $EMST$ and return a DFS traversal tour.
- Complexity?

Computing the EMST

- What is the best algorithm to compute an $EMST$?
- Using Prim's/Kruskal's algorithm the complexity will be $O(n^2)$.
  - Why?
- Can we do better?
- Recall that $EMST \subseteq DT$
- Compute the DT of the set of points - $O(n \log n)$
- Compute the $EMST$ of the DT using Prim's/Kruskal's algorithm - $O(n \log n)$
- Total complexity - $O(n \log n)$